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EM-D Stream Mathematical Supplement

Complete derivations for the field topology, momentum transfer, radiation bound, circuit condition, and hull loading.

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1 Notation and constants

SYMBOL	QUANTITY	VALUE / UNITS
μ_0	vacuum permeability	4π×10^{-7} H/m
c	speed of light	2.99792458×10^{8} m/s
m_p	proton mass	1.67262192×10^{-27} kg
m	magnetic dipole moment	A·m²
R	dynamo ring radius	m
n, v	solar wind density, speed (1 AU)	5×10^{6} m^{-3}, 400 km/s
η	radiated momentum fraction	dimensionless, ≤ 1

2 Field-reversed core: the null condition

The central topological result. N identical dipoles of moment $m\,\hat{\mathbf{z}}$ sit on a ring of radius R in the plane $z = 0$.

STEP 1 — DIPOLE FIELD

The field of a point dipole $\mathbf{m}$ at displacement $\mathbf{r}$ is

$$B(\mathbf{r}) = (\mu_0/4\pi) \cdot [\, 3\hat{\mathbf{r}}(\mathbf{m}\cdot\hat{\mathbf{r}}) - \mathbf{m}\,] \,/\, r^3 \tag{2.1}$$

STEP 2 — GEOMETRY ON THE AXIS

For a field point at (0, 0, z) and a source at radius R in the plane, the separation is $\mathbf{d} = (-R\cos\varphi, -R\sin\varphi, z)$, so

$$r = \sqrt{(R^2 + z^2)} \qquad \mathbf{m}\cdot\hat{\mathbf{r}} = m\,z \,/\, r \tag{2.2}$$

STEP 3 — TRANSVERSE CANCELLATION

Summing over φ uniformly distributed, the x and y components cancel by symmetry for $N \geq 2$. Only B_z survives.

$$B_z(z) = N\,(\mu_0 m/4\pi) \cdot (3z^2 - r^2) \,/\, r^5 \tag{2.3}$$

STEP 4 — SUBSTITUTE R^2

$$B_z(z) = N\,(\mu_0 m/4\pi) \cdot (2z^2 - R^2) \,/\, (R^2 + z^2)^{5/2} \tag{2.4}$$

STEP 5 — THE NULL

The denominator is strictly positive, so $B_z = 0$ requires $2z^2 = R^2$:

$$z_{null} = \pm\, R \,/\, \sqrt{2} \tag{2.5}$$

CONSEQUENCE

Equation (2.5) contains neither m nor N. **Current sets field strength; geometry sets field topology.** No increase in drive current moves the nulls outward. In vacuum the topological scale of the machine equals its hardware scale — a field structure larger than the hardware requires an external current carrier.

Verification. For R = 10 m, (2.5) predicts ±7.0711 m. Numerical root-finding on a 1201-point axial sample of the summed field locates sign changes at ±7.0711 m. Between the nulls $B_z < 0$: the core field is reversed relative to the moments producing it, which is a field-reversed configuration.

3 Why momentum transfer requires a translating pattern

Induction couples to the plasma through the electric field, which follows from the vector potential:

$$\mathbf{A}(\mathbf{r},t) = \Sigma_i\,(\mu_0/4\pi)\,\mathbf{m}_i(t) \times (\mathbf{r}-\mathbf{r}_i)\,/\,|\mathbf{r}-\mathbf{r}_i|^3 \qquad \mathbf{E} = -\partial\mathbf{A}/\partial t \tag{3.1}$$

Consider uniform amplitude modulation, $\mathbf{m}_i(t) = f(t)\,\mathbf{m}_i(0)$. Since (3.1) is linear in each moment:

$$\mathbf{E} = -(\mathrm{d}f/\mathrm{d}t)\,\mathbf{A}_{\mathrm{unit}}(\mathbf{r}) \tag{3.2}$$

The vector potential of an axial dipole is purely **azimuthal**: $\mathbf{m} \times \mathbf{r} \propto \hat{\boldsymbol{\varphi}}$ for $\mathbf{m} \parallel \hat{\mathbf{z}}$. Therefore $\mathbf{E} \parallel \hat{\boldsymbol{\varphi}}$ everywhere, and the force per unit charge has no axial component at any point.

RESULT

A breathing mode drives azimuthal current and imparts *angular* momentum only. Geometric asymmetry cannot repair this, because the asymmetry resides in **B** while the momentum transfer is performed by **E**. Axial thrust requires the field *pattern* to translate — that is, sources fired in temporal sequence, giving each $f_i(t)$ an independent phase.

Measured. Boris-pusher integration including both **E** and **B**, 2500 protons, $\mathrm{d}t = 5\times10^{-8}$ s:

DRIVE	ENSEMBLE ΔP_z (KG·M/S)	SIGNIFICANCE
Symmetric control, 5 seeds	$+4.27\times10^{-21}$	**0.47σ — pass**
Zero delay (breathing)	$+1.33\times10^{-20}$	2.0σ — null
Wave toward $-z$	$+5.42\times10^{-20}$	**8.1σ**
Wave toward $+z$	-4.71×10^{-20}	**7.0σ**

Noise floor taken as the random-walk value $\sqrt{N}\cdot m\cdot v_{\mathrm{th}}$. Using $N\cdot m\cdot v_{\mathrm{th}}$ would understate it by $\sqrt{N} = 50$ and convert noise into apparent signal.

4 The radiation bound

For any radiating system, the momentum carried by the electromagnetic field per unit energy is exactly $1/c$. Defining the momentum fraction over the far-field intensity $I(\hat{\mathbf{n}})$:

$$\eta \equiv | \oint I(\hat{\mathbf{n}})\, \hat{\mathbf{n}}\, d\Omega \,| \,/ \oint I(\hat{\mathbf{n}})\, d\Omega \qquad F = \eta\, P_{rad} \,/\, c \tag{4.1}$$

Since $\eta \leq 1$ by the triangle inequality:

$$F \leq P/c = 3.336{\times}10^{-6}\ \text{N per kW} \tag{4.2}$$

4.1 Why a bare aperture gives exactly zero

A uniformly illuminated elliptical aperture with semi-axes a, b has far field

$$E(\theta,\varphi) \propto 2J_1(u)/u, \quad u = k\sqrt{[(a \sin\theta\cos\varphi)^2 + (b \sin\theta\sin\varphi)^2]} \tag{4.3}$$

Since u depends on θ only through $\sin\theta$, the pattern satisfies $I(\hat{\mathbf{n}}) = I(-\hat{\mathbf{n}})$. The integrand in (4.1) is odd and the integral vanishes identically: $\eta = 0$.

4.2 The reflector

Suppressing the rear hemisphere by power ratio r and splitting (4.1) into hemispheres of equal forward and backward flux I_0:

$$\eta = (I_0 - rI_0) \,/\, (I_0 + rI_0) = (1 - r)/(1 + r) \tag{4.4}$$

FRONT-TO-BACK	*R*	*H* FROM (4.4)	COMPUTED
0 dB	1.000	0.000	**0.00000**
10 dB	0.100	0.818	0.79916
20 dB	0.010	0.980	0.95741
30 dB	0.001	0.998	**0.97480**

The computed values fall slightly below (4.4) because rim spillover is not captured by a pure two-hemisphere split; the residual at 40 dB (0.97656) is spillover-limited, not backlobe-limited.

4.3 Conversion to electrical power

$$F = \eta_{\text{beam}} \cdot \eta_{\text{conv}} \cdot P_{\text{elec}} / c \tag{4.5}$$

With $\eta_{\text{beam}} = 0.9748$ and a magnetron at $\eta_{\text{conv}} = 0.85$, the delivered figure is **2.764×10^{-6} N per kW electrical**.

5 The circuit condition

Force on a current-carrying conductor in an external field:

$$\mathbf{F} = I \oint \mathrm{d}\mathbf{l} \times \mathbf{B} \tag{5.1}$$

For uniform **B**, the field is constant across the integral:

$$\mathbf{F} = I \left(\oint \mathrm{d}\mathbf{l} \right) \times \mathbf{B} = I \cdot \mathbf{0} \times \mathbf{B} = \mathbf{0} \tag{5.2}$$

because the closed-path integral of the displacement element vanishes identically. The torque, however, does not:

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}, \quad \mathbf{m} = I\mathbf{A} \tag{5.3}$$

RESULT

A closed loop in a uniform field receives a torque and no net force — it aligns like a compass needle without translating. Net force requires either a field *gradient*, $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$, which at the interplanetary gradient $B/\mathrm{AU} = 3.34\times10^{-20}$ T/m yields 10^{-10} N for a 100 m coil at 100 kA; or an **open** conductor whose circuit closes externally. In space the only available external conductor is ambient plasma. *That is the entire plasma requirement* — a current needs a return path, and vacuum has infinite resistance.

Verified numerically by direct integration of (5.1) over 20 000 segments at loop radii 1–20 m and currents 1–100 kA: net force at machine precision (10^{-21} to 10^{-18} N) in every case.

6 Magnetic sail: standoff and thrust

The standoff radius is defined by pressure balance between the vehicle field and the solar wind ram pressure:

$$B(r_{\mathrm{mp}})^2 \,/\, 2\mu_0 = \rho v^2 \quad \text{with} \quad B(r) = \mu_0 m \,/\, 4\pi r^3 \tag{6.1}$$

Solving for r_{mp}:

$$r_{mp} = [\, \mu_o m^2 / (32\pi^2 \rho v^2) \,]^{1/6} \tag{6.2}$$

The intercepted momentum flux gives the thrust:

$$F = C_d \cdot \rho v^2 \cdot \pi r_{mp}^2 \tag{6.3}$$

CORRECTION — 134×

An earlier revision computed (6.3) using the *interplanetary field's magnetic pressure*, $B_{IMF}^2/2\mu_o$ = 9.947×10^{-12} Pa, rather than the wind's *ram* pressure, $\rho v^2 = 1.338\times10^{-9}$ Pa. Equation (6.1) makes the correct choice explicit: at the standoff radius the two pressures are equal *by definition of* r_{mp}, and the momentum delivered is the ram flux. The ratio is 134.5, and every thrust and transit figure preceding Rev C was understated by that factor.

BUBBLE RADIUS	MOMENT REQUIRED	THRUST (6.3)	MARS TRANSIT, 37.4 T
30 km	1.4×10^{13}	3.78 N	641 d
60 km	1.1×10^{14}	15.1 N	160 d
120 km	1.00×10^{15}	**60.5 N**	**40 d**
300 km	1.57×10^{16}	378 N	6 d

7 Hull loading

The Maxwell stress tensor for a magnetic field:

$$T_{ij} = (1/\mu_0)\,(\,B_iB_j - \tfrac{1}{2}\,\delta_{ij}B^2\,) \tag{7.1}$$

The traction on a surface element with outward normal $\hat{\mathbf{n}}$ is $\mathbf{t} = \mathbf{T}\cdot\hat{\mathbf{n}}$, carrying tension $B^2/2\mu_0$ along the field and equal pressure across it. Integrating over a closed surface enclosing no sources gives zero by Gauss's theorem, since $\nabla\cdot\mathbf{T} = 0$ in a source-free static region.

Verified. Grid convergence of the residual on the hull sphere, four dynamos at 40° cant:

GRID (N_Θ)	RESIDUAL |F| (N)	RATIO
60	1.1573×10^{-2}	—
120	2.8889×10^{-3}	4.01
240	7.2194×10^{-4}	4.00
480	1.8047×10^{-4}	4.00

Exactly second-order convergence to zero, confirming the residual is quadrature error rather than force. Forward and aft hemispheres carry equal and opposite axial traction to five significant figures at every cant angle: **the hull is stretched, not propelled.**

7.1 Structural limit

Peak traction scales as m^2, and thin-wall hoop stress is $\sigma = pR/2$:

$$\sigma(m) = \sigma_0\,(m/m_0)^2 \tag{7.2}$$

MOMENT PER DYNAMO	HOOP STRESS	FRACTION OF AL YIELD
1×10^6 A·m²	0.04 MPa	0.01%
1×10^7 A·m²	3.69 MPa	1.4%
1×10^8 A·m²	369 MPa	137%

Practical ceiling for a 2.2 m aluminium hull: **≈3×10^7 A·m² per dynamo**.

8 Vehicle

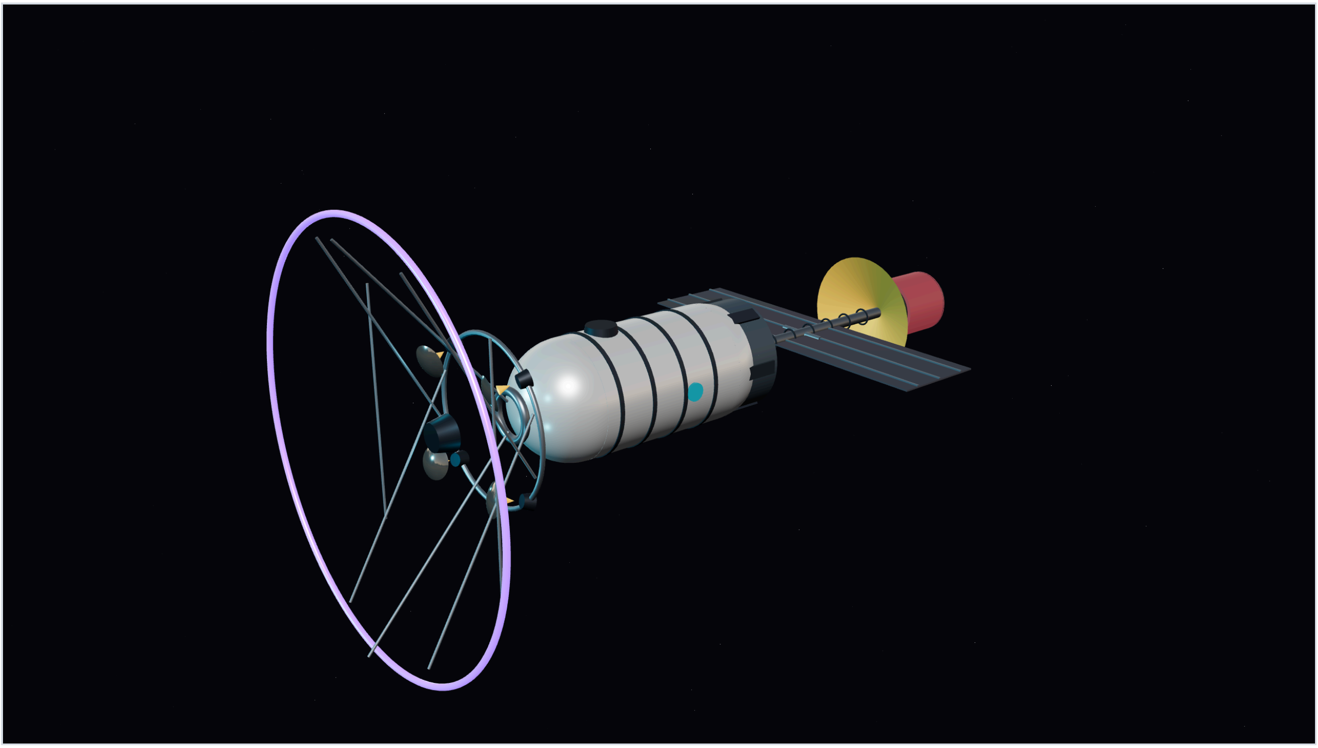

Figure M1. Crewed configuration, deployed. Forward: 15 m superconducting coil on eight spars, then the four reflector-fed dynamos on a 3 m ring at 20° cant. Centre: 4.4 m × 6 m habitat with service section. Aft: 24 m² radiators, 12 m boom, shadow shield, 100 kW_e reactor. Geometry to the values in §6 and §7.

Derivations verified against: `emd_rubberband.py` (§2), `emd_vectoring.py` (§3), `emd_aperture.py` (§4), `emd_hull.py` (§5), `emd_corrected.py` (§6), `emd_angles.py` (§7).